647510 **BSc5MatC503x** Seat No :________

B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
March/April-2025 (NEW COURSE)
Boolean Algebra & Complex Analysis -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1 **(A) Answer any two:** **(10)**

1) Let N denote the set of all natural numbers and R be the relation on $N \times N$, defined by $(a, b)\ R\ (c, d) \iff ad(b + c) = bc(a + d)$, check whether R is an equivalence relation on $N \times N$.
2) Define: Lattice as a partial ordered set. State and prove distributive inequalities in lattice.
3) If $(L,* \oplus)$ is a lattice and R is partial order relation in L defined as $a\ R\ b \iff a * b = a$ then for any $a, b \in L$, prove that: $GLB\{a, b\} = a * b$ and $LUB\{a, b\} = a \oplus b$.

(B) Answer any one: **(04)**

1) Define: cover of an element of a poset. Draw the Hasse diagram of $(P(A), \subset), where\ A = \{a, b, c\}$.
2) Let Z be the set of integers. Define $R = \{(a, b): b = a^r, for\ some\ positive\ integer\ r\}$; show that (Z, R) is a partial order set.

Que-2 **(A) Answer any two:** **(10)**

1) Define: Boolean algebra. In usual notation, show that $(S_6,\ *,\ \oplus, ', 0,1)$ is a Boolean algebra.
2) Define: Cube array representation of Boolean algebra. Find the cube array representation of $f(x, y, z) = xy + xz'$.
3) State and prove Stone's representation theorem for Boolean algebra.

(B) Answer any one: **(04)**

1) Prove that the sum of all minterms of n variables $x_1, x_2, \dots x_n$ is 1.
2) Express the Boolean expression $\alpha(x_1, x_2, x_3) = x_2 * x_3$ as the product of sum canonical form.

Que-3 **(A) Answer any two:** **(10)**

1) Show that the function $f(z) = \frac{(\bar{z})^2}{z}, z \neq 0$
$= 0\ , z = 0$

satisfies C-R equations at origin but it is not differentiable at origin.
2) State and prove necessary and sufficient condition for a function to be analytic.
3) Find an analytic function $f(z) = u + iv$ such that
$$u - v = (x^3 - y^3)(x - y) - 6x^2y^2.$$

(B) Answer any one: **(04)**

1) Show that $u = 2x(1 - y)$ is harmonic function and find its corresponding conjugate harmonic function.
2) Find an analytic function whose real part is $cosxcoshy$.

Que-4 **(A) Answer any two:** **(10)**

1) State and prove Cauchy's integral formula.
2) In usual notation prove that: $\left|\int_a^b f(z)dz\right| \leq \int_a^b |f(z)|dz$.
3) Define: Closed curve, prove that $\int_c \ \pi e^{\pi \bar{z}} dz = 4(e^{\pi} - 1)$, where c is the square joining points $z = 0, z = 1, z = 1 + i$ and $z = i$.

(B) Answer any one: **(04)**

1) Find: $\int_0^{2+i} (\bar{z})^2 dz$
2) Show that $\left|\int_c \ \frac{z+4}{z^3-1} dz\right| \leq \frac{6\pi}{7}$, where c be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$.

Que-5 **(A) Answer any two:** **(10)**

1) Prove that non-constant entire functions are unbounded in C.
2) State Cauchy's integral formula for derivatives and using it find: $\int_c \ \frac{3z^2-7z+1}{(z-1)^3} dz$ where $c: |z| = 3$.
3) Prove that any polynomial over C of degree $n, (n \geq 1)$ has exactly n roots.

(B) Answer any one: **(04)**

1) If $g(z_0) = \int_c \ \frac{z^2+2z+5}{z-z_0} dz, c: |z| = 3$ then find $g(0), g(1), g'(-2), g(i)$ and $g(z_0)$ when $|z_0| > 3$.
2) State and prove Cauchy's inequality.

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